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CP Violation and Strong Phases from Penguins in $B^{\pm} \to VV$ Decays

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Abstract

We calculate direct CP-violating observables in charged $B \to VV$ decays arising from the interference of amplitudes with different strong and CKM phases. The perturbative strong phases develop at order α_s from absorptive parts of one-loop matrix elements of the next-to-leading logarithm corrected effective Hamiltonian. CPT constraints are maintained. Based on this model, we find that partial rate asymmetries between charge conjugate $B^{\pm}$ decays can be as high as 15-30% for certain channels with branching ratios in the 10^{-6} range. The small values of the coefficients of angular correlations, which we calculated previously to be of order 10^{-2}, are not significantly degraded by the strong phases. The charge asymmetries of rates and angular distributions would provide unambiguous evidence for direct CP violation.

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1 Introduction

So far CP violation [1] has been detected only in processes related to $K^0 - \bar{K}^0$ mixing [2] but considerable efforts are being made to investigate it in B decays. While the most promising proposal for observing CP violation in the B system involves the mixing between neutral B mesons [3], the particular interest in decays of charged B mesons lies in their possibilities for establishing the detailed nature of CP violation. Since charged B mesons can not mix, a measurement of a CP violating observable in these decays would be a clear sign for *direct* CP violation, which has been searched for in K decays with indefinite success as the measurements of ϵ'/ϵ do not yet exclude a zero value [4].

In non-leptonic charged B decays two main categories of CP-violating observables can be investigated: First, rate asymmetries [5, 6],

$$a_{CP} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \ , \tag{1}$$

where Γ and $\bar{\Gamma}$ are the (partial) rates of the decay and its charge (C) conjugate, and second, azimuthal angular correlations [7, 8].

The rate asymmetries occur even for spinless final states and require both weak *and* strong phase differences in interfering amplitudes. The weak phase differences arise from the superposition of amplitudes from various penguin diagrams and the usual W-exchange (if contributing). The strong phase is generated by final state interactions. At the quark level these strong interaction effects can be modeled by absorptive parts of perturbative penguin diagrams (hard final state interactions) [5] while predictions at the non-perturbative hadronic level are of course extremely difficult (soft final state interactions). Clearly we can not exclude that the weak transition matrix elements receive phases originating from soft final state interactions (resonances) between the produced vector particles. However, since the mass of the B is far above the usual resonance region, we expect these phase shifts to be small.

It is clear that a significant contribution of penguin diagrams, and hence of the CKM [9] phase differences, is an exceptional case and requires either the absence or a strong CKM suppression of the tree contributions (as e.g. in charmless $b \to s$ transitions).

The rate asymmetries for exclusive two-body decays into pseudoscalars have been estimated by several authors using either the model of Bauer, Stech and Wirbel [10] (BSW) based on wave functions in the infinite momentum frame, or the perturbative methods developed by Brodsky et. al. [11]. The rate asymmetries a_{CP} can be quite large (of the order $a_{CP} \sim 0.1$) for some of the final states. However, the corresponding branching fractions of these decays are quite small, ranging from 10^{-6} (estimates with the BSW model [10]) to 10^{-7} (estimates with the Brodsky-Lepage model [11, 12, 28]). The magnitude of $a_{CP}^2 \times BR$ is therefore of the order of $10^{-9} - 10^{-7}$.

The second category of CP-violating observables involves the decay of the B meson into two vector particles $B \to V_1 V_2$ with subsequent decays of V_1 and V_2 [7, 8]. (In the following B will always denote the B^- meson and $\bar{B}$ its antiparticle.) By analyzing the azimuthal dependence of the vector meson decay products one can then isolate CP odd quantities. The advantage of this method is that the CP violating terms occur even when there are no strong phase differences between the interfering weak amplitudes. On the other hand these coefficients in the azimuthal correlation are also present when the CP-violating weak phase differences vanish. By measuring these coefficients in charge conjugate $B^\pm$ decays one has the possibility to disentangle the effects of strong and weak phases [8].

So far the "CP-odd" azimuthal angular coefficients have been calculated under the assumption that the strong phase differences vanish [8]. These results give us an estimate of the effect we could expect from the CP-violating phase. Since strong phase differences are present, at least due to the hard final state interactions which lead to the rate asymmetries through the Bander-Silverman-Soni mechanism [5], we should include these strong phases also in the calculation of the "CP-odd" angular coefficients.

The two categories for detecting direct CP violation are therefore complementary. The rate asymmetries occur only when non-vanishing strong phase differences are present but need no angular information. The azimuthal angular correlation terms need no strong phase differences but require joint angular distribution measurements of the decay products of the B mesons [14].

Although the explicit form of these angular distributions depends on the spins of the decay products of the decaying vector mesons V_1 and V_2, two formulas are sufficient to describe a general B→VV decay [8]. For instance, the angular distribution for the cascade decay $B^- \to K^{*-}\rho^0 \to (K\pi)\,(\pi^+\pi^-)$ has the following form:

$$
\begin{aligned}
\frac{d^3\Gamma}{dcos\theta_1 dcos\theta_2 d\phi} \sim \; & \frac{1}{4}\frac{\Gamma_T}{\Gamma} \cdot sin^2\theta_1 \; sin^2\theta_2 + \frac{\Gamma_L}{\Gamma} \cdot cos^2\theta_1 \; cos^2\theta_2 \\
& + \frac{1}{4} \,\mathrm{sin}2\theta_1 \; sin2\theta_2 [\alpha_1 \cdot cos\phi - \beta_1 \cdot sin\phi] \\
& + \frac{1}{2} sin^2\theta_1 sin^2\theta_2 \; [\alpha_2 \cdot cos2\phi \; - \beta_2 \cdot sin2\phi \;] \; .
\end{aligned}
\tag{2}
$$

In eq. (2) θ_1 is the polar angle of the K in the rest system of the K^* with respect to the helicity axis. Similarly θ_2 and ϕ are the polar and azimuthal angle of the π^+ in the ρ^0 rest system with respect to the helicity axis of the ρ^0; i.e. ϕ is the angle between the planes of the two decays $K^{*-} \to K\pi$ and $\rho^0 \to \pi^+\pi^-$.

The decay distribution is parameterized by the coefficients:

$$
\begin{array}{rclrcl}
\frac{\Gamma_T}{\Gamma} & = & \frac{|H_{+1}|^2+|H_{-1}|^2}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2} & \frac{\Gamma_L}{\Gamma} & = & \frac{|H_0|^2}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2} \\
\alpha_1 & = & \frac{Re\left(H_{+1}H_0^*+H_{-1}H_0^*\right)}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2} & \beta_1 & = & \frac{Im\left(H_{+1}H_0^*-H_{-1}H_0^*\right)}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2} \\
\alpha_2 & = & \frac{Re\left(H_{+1}H_{-1}^*\right)}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2} & \beta_2 & = & \frac{Im\left(H_{+1}H_{-1}^*\right)}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2}
\end{array} \tag{3}
$$

where $H_\lambda = \langle V_1(\lambda)V_2(\lambda)|\mathcal{H}_{\text{eff}}|\bar{B}\rangle$ are the helicity amplitudes ($\lambda = 0, \pm 1$). Clearly eq. (2) can be used also for all other decays where V_1 and V_2 decay into two pseudo scalar mesons. The other decay distribution, e.g. for $B \to D_s^* D^*$ with $D_s^* \to D_s\gamma$ and $D^* \to D\pi$ can be found in ref. [8], where further examples are discussed.

In our previous work, we have calculated the six angular coefficients and the branching ratios for 36 decays $B \to V_1V_2$ with neutral and charged B mesons. Penguin contributions were taken into account only through leading-log short distance QCD effects and all strong phases were neglected. Non-vanishing "CP-odd" azimuthal angular correlations, i.e. coefficients β_1 and β_2 occurred only in the decays $B^- \to K^{*-}\omega$, $K^{*-}\rho^0$, $\omega\rho^-$ (and the corresponding decays of $\bar{B}^0$). In this work, we shall present results including strong phases from penguin diagram contributions to the matrix elements. We consider the rate asymmetries a_{CP} and the decay parameters from eq. (3) for B^- and B^+ decays, and investigate the influence of the strong phases on $\beta_{1,2}$. In addition, we base our treatment on the next-to-leading logarithmic short distance corrections evaluated by Buras et al. [15], which is mandatory if one wants to systematically take into account the complete $O(\alpha_s)$ penguin matrix elements. We include also some pure penguin modes which are of interest for the detection of CP effects via the rate asymmetry a_{CP}, and we give estimates of their branching ratios.

The remainder of this paper is organized as follows. In section 2 we describe the effective weak Hamiltonian and the evaluation of the hadronic matrix elements. The CP-violating observables are discussed in section 3. The final results for the angular correlations and rate differences are discussed in section 4. Formulae for the matrix elements and some technical details about CPT cancellations can be found in the appendices.

2 The effective Hamiltonian

2.1 Short distance QCD corrections

For calculations of CP-violating observables it is most convenient to split the effective weak Hamiltonian into two pieces, one proportional to $v_u \equiv V_{ub}V_{us}^*$ (or $V_{ub}V_{ud}^*$ in the

case of $b \to d$ transitions) and the other one proportional to $v_c \equiv V_{cb}V_{cs}^*$ (or $V_{cb}V_{cd}^*$ correspondingly),

$$\mathcal{H}_{\rm eff} = 4\frac{G_F}{\sqrt{2}}\left(v_u \mathcal{H}_{\rm eff}^{(u)} + v_c \mathcal{H}_{\rm eff}^{(c)}\right) \ . \tag{4}$$

The two terms ($q = u, c$)

$$\mathcal{H}_{\rm eff}^{(q)} = \sum_i c_i(\mu) \cdot O_i^{(q)} \ ,$$

differ only by the quark content of the local operators, and for our purposes it is sufficient to consider only the following four-quark operators [16]:

$$\begin{array}{rclrcl} O_1^{(q)} &=& \bar{s}_\alpha \gamma^\mu L q_\beta \cdot \bar{q}_\beta \gamma_\mu L b_\alpha \ , & O_2^{(q)} &=& \bar{s}_\alpha \gamma^\mu L q_\alpha \cdot \bar{q}_\beta \gamma_\mu L b_\beta \ , \\ O_3 &=& \bar{s}_\alpha \gamma^\mu L b_\alpha \cdot \sum_{q'} \bar{q}'_\beta \gamma_\mu L q'_\beta \ , & O_4 &=& \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \sum_{q'} \bar{q}'_\beta \gamma_\mu L q'_\alpha \ , \\ O_5 &=& \bar{s}_\alpha \gamma^\mu L b_\alpha \cdot \sum_{q'} \bar{q}'_\beta \gamma_\mu R q'_\beta \ , & O_6 &=& \bar{s}_\alpha \gamma^\mu L b_\beta \cdot \sum_{q'} \bar{q}'_\beta \gamma_\mu R q'_\alpha \ . \end{array} \tag{5}$$

where L and R are the left- and right-handed projection operators. The operators $O_3, \ldots, O_6$ arise from (QCD) penguin diagrams which enter at order α_s in the initial values of the coefficients,

$$c_i(M_W) = \begin{cases} 1 + O(\alpha_s) & (i = 2) \\ O(\alpha_s) & (\text{otherwise}) \end{cases} \ ,$$

or through operator mixing during the renormalization group summation of short distance QCD corrections. The renormalization group evolution from $\mu \approx M_W$ to $\mu \approx m_b$ has been evaluated in next-to-leading logarithmic (NLL) precision by Buras et al. [15]. These authors also demonstrated how the $O(\alpha_s)$ renormalization scheme dependence can be isolated in terms of a matrix $\mathbf{r}_{ji}$ by writing

$$c_j(\mu) = \sum_i \bar{c}_i(\mu) \left[\delta_{ij} - \frac{\alpha_s(\mu)}{4\pi}\mathbf{r}_{ij}\right] \ , \tag{6}$$

where the coefficients $\bar{c}_j$ are scheme independent at this order. The numerical values for $\Lambda^{(4)}_{\overline{MS}} = 350$ MeV [3], $m_t = 150$ GeV and $\mu = m_b = 4.8$GeV are [15]

$$\begin{array}{rclrcl} \bar{c}_1 &=& -0.324 \ , & \bar{c}_2 &=& 1.151 \ , \\ \bar{c}_3 &=& 0.017 \ , & \bar{c}_4 &=& -0.038 \ , \\ \bar{c}_5 &=& 0.011 \ , & \bar{c}_6 &=& -0.047 \ . \end{array} \tag{7}$$

Contributions from the color magnetic moment operator

$$O_g = \frac{g_s}{16\pi^2} \cdot \Psi \sigma_{\mu\nu} \left(m_b R + m_s L\right) \frac{\lambda^a}{2} \Psi \ G_a^{\mu\nu} \ ,$$

[3] This value of $\Lambda^{(4)}_{\overline{MS}}$ translates to $\Lambda^{(5)}_{\overline{MS}} \approx 250 MeV$, which is about the value from a recent compilation of G. Altarelli ($\Lambda^{(5)}_{\overline{MS}} = 240 \pm 90 MeV$ [17])

with a coefficient of the order -0.15, will allways be neglected in the following, because already its tree level matrix elements are suppressed by a factor $\alpha_s/4\pi$ and it cannot provide interesting absorptive parts in the decays considered here.

2.2 Quark-level matrix elements

Working at NLL precision, it is consistent – and necessary in order to cancel the scheme dependence from the renormalization group evolution – to treat the matrix elements of $\mathcal{H}_{\rm eff}$ at the one-loop level. These one-loop matrix elements can be rewritten in terms of the tree-level matrix elements of the effective operators, and one obtains:

$$\langle sq'\bar{q}'|\mathcal{H}_{\rm eff}^{(q)}|b\rangle = \sum_{i,j} c_i(\mu)\left[\delta_{ij} + \frac{\alpha_s(\mu)}{4\pi}\mathbf{m}_{ij}(\mu,\ldots)\right]\langle sq'\bar{q}'|O_j^{(q)}|b\rangle^{\rm tree}\ . \tag{8}$$

The functions $\mathbf{m}_{ij}$ are determined by the corresponding renormalized one-loop diagrams and depend in general on the scale μ, on the quark masses and momenta, *and* on the renormalization scheme. The various one-loop diagrams can be grouped into two classes: *vertex-corrections*, where a gluon connects two of the outgoing quark lines (fig. 1a), and *penguin* diagrams, where a quark-antiquark line closes a loop and emits a gluon, which itself decays finally into a quark-antiquark pair (fig. 1b).

When expressing the rhs of eq. (8) in terms of the renormalization scheme independent coefficients $\bar{c}_i$, the effective coefficients multiplying the matrix elements $\langle sq'\bar{q}'|O_j^{(q)}|b\rangle^{\rm tree}$ become

$$c_j^{\rm eff} \equiv \bar{c}_j + \frac{\alpha_s}{4\pi}\sum_i \bar{c}_i\cdot(\mathbf{m}_{ij}-\mathbf{r}_{ij})\ . \tag{9}$$

The renormalization scheme dependence, which is present in $\mathbf{m}_{ij}$ and $\mathbf{r}_{ij}$, explicitly cancels in the combination $\mathbf{m}_{ij}-\mathbf{r}_{ij}$. This reflects the familiar fact that one-loop matrix elements have to be included in order to compensate for the order α_s renormalization scheme dependence which enters through the coefficients c_i in $\mathcal{H}_{\rm eff}$ (generated by the NLL renormalization group evolution). For instance, the scheme dependence in the coefficients of the penguin operators, which enter via $c_j\langle sq'\bar{q}'|O_j^{(q)}|b\rangle^{\rm tree}$ for $j=3,4,5,6$ in eq. (8), is cancelled by penguin-like matrix elements (fig. 1b) of the operators.

The effective coefficients $c_{1,2}^{\rm eff}$ receive contributions only from *vertex-correction* diagrams, which will not be included in the following (see the discussion in Section 3). For a general $SU(N)$ color group the remaining effective coefficients can be brought into the following form

$$\begin{aligned} c_3^{\rm eff} &= \bar{c}_3 - \frac{1}{2N}\frac{\alpha_s}{4\pi}(c_t+c_p)+\cdots \\ c_4^{\rm eff} &= \bar{c}_4 + \frac{1}{2}\frac{\alpha_s}{4\pi}(c_t+c_p)+\cdots \end{aligned}$$

$$
\begin{aligned}
c_5^{\rm eff} &= \bar{c}_5 - \frac{1}{2N}\frac{\alpha_s}{4\pi}(c_t + c_p) + \cdots \\
c_6^{\rm eff} &= \bar{c}_6 + \frac{1}{2}\frac{\alpha_s}{4\pi}(c_t + c_p) + \cdots \ ,
\end{aligned} \tag{10}
$$

where we have separated the contributions c_t and c_p from the "tree" operators $O_{1,2}$ and from the penguin operators $O_{3\cdots 6}$, respectively. The ellipses denote further contributions from *vertex-correction* diagrams.

In addition to the contributions from penguin diagrams with insertions of the tree operators $O_{1,2}^{(q)}$

$$
c_t = \bar{c}_2 \cdot \left[\frac{10}{9} + \frac{2}{3}\ell n\frac{m_q^2}{\mu^2} - \Delta F_1\Big(\frac{k^2}{m_q^2}\Big)\right] \ , \tag{11}
$$

where ΔF_1 is defined in appendix A, we have evaluated the penguin diagrams for the matrix elements of the penguin operators:

$$
\begin{aligned}
c_p &= \bar{c}_3 \cdot \left[\frac{280}{9} + \frac{2}{3}\ell n\frac{m_s^2}{\mu^2} + \frac{2}{3}\ell n\frac{m_b^2}{\mu^2} - \Delta F_1\Big(\frac{k^2}{m_s^2}\Big) - \Delta F_1\Big(\frac{k^2}{m_b^2}\Big)\right] \\
&+ (\bar{c}_4 + \bar{c}_6) \cdot \sum_{j=u,d,s,\ldots} \left[\frac{10}{9} + \frac{2}{3}\ell n\frac{m_j^2}{\mu^2} - \Delta F_1\Big(\frac{k^2}{m_j^2}\Big)\right] \ ,
\end{aligned} \tag{12}
$$

2.3 Hadronic matrix elements in the BSW model

In order to evaluate the hadronic matrix elements of $\mathcal{H}_{\rm eff}$ we represent the helicity amplitudes in terms of three invariant amplitudes, a, b and c:

$$
H_\lambda \equiv \epsilon_{1\mu}(\lambda)^* \epsilon_{2\nu}(\lambda)^* \left[a g^{\mu\nu} + \frac{b}{m_1 m_2}\ p_2^\mu p_1^\nu + \frac{ic}{m_1 m_2}\epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta}\right] \ , \tag{13}
$$

where $p_{1,2}$ and $m_{1,2}$ are the four-momenta and masses of $V_{1,2}$, respectively.

The coefficients a, b and c have strong phases δ from final state interactions (e.g. between the two vector particles V_1 and V_2) and weak phases ϕ originating from the CP violating phase in the CKM matrix. In general, the invariant amplitudes are a sum of several interfering amplitudes, a_k, b_k, and c_k, respectively, corresponding to, for instance, various different isospin contributions. Then the phase structure of a, b and c is:

$$
\begin{aligned}
a &= \sum_k |a_k| e^{i\delta_{a_k} + i\phi_{a_k}} \\
b &= \sum_k |b_k| e^{i\delta_{b_k} + i\phi_{b_k}} \\
c &= \sum_k |c_k| e^{i\delta_{c_k} + i\phi_{c_k}} \ .
\end{aligned} \tag{14}
$$

The helicity amplitudes $\bar{H}_\lambda$ for the decay of $\bar{B} \to \bar{V}_1 \bar{V}_2$, where $\bar{V}_1$ and $\bar{V}_2$ are the antiparticles of V_1 and V_2 respectively have the same decomposition as (12) with $a \to$

$\tilde{a}, b \to \tilde{b}$ and $c \to -\tilde{c}$. The amplitudes $\tilde{a}, \tilde{b}$ and $\tilde{c}$ have the analogous phase structure as above with the weak phases changing sign $\phi_{a_k,b_k,c_k} \to -\phi_{a_k,b_k,c_k}$; i.e. the $\tilde{a}_k$, $\tilde{b}_k$ and $\tilde{c}_k$ have the same strong phase shifts and the opposite weak phase compared to a_k, b_k and c_k.

From the decomposition eq. (13) one finds the following relations between the helicity amplitudes and the invariant amplitudes, a, b, c:

$$H_{\pm 1} = a \pm c\sqrt{x^2 - 1} \quad \text{and} \quad H_0 = -ax - b\left(x^2 - 1\right) \ , \tag{15}$$

where

$$x = \frac{p_1 p_2}{m_1 m_2} = \frac{m_B^2 - m_1^2 - m_2^2}{2 m_1 m_2} \ .$$

When there are no strong interaction phases, one has $\tilde{a} = a^*$, $\tilde{b} = b^*$ and $\tilde{c} = c^*$. Due to the sign change in front of $\tilde{c}$ in $\bar{H}_\lambda$, we have in this case

$$\bar{H}_{\pm 1} = H^*_{\mp 1} \ , \quad \bar{H}_0 = H^*_0 \ .$$

To take into account long distance QCD effects which build up the hadronic final states, we follow Bauer, Stech and Wirbel [10]: With the help of the factorization hypothesis [18] the three-hadron matrix elements are split into vacuum-meson and meson-meson matrix elements of the quark currents entering in $O_1, \ldots, O_6$. In addition, OZI suppressed form factors and annihilation terms are neglected. In the BSW model, the meson-meson matrix elements of the currents are evaluated by overlap integrals of the corresponding wave functions and the dependence on the momentum transfer (which is equal to the mass of the factorized meson) is modeled by a single-pole ansatz. As a first approximation, this calculational scheme provides a reasonable method for estimating the relative size and phase of the tree and penguin terms that give rise to the CP-violating signals.

Concerning the QCD coefficients and how $1/N$ terms are treated, it is well known [19] that this model has problems accounting for the decays with branching ratios which are proportional to a_2^2, where

$$a_2 = \bar{c}_1 + \frac{1}{N}\bar{c}_2 \ .$$

This is due to the fact that a_2 has a rather small value $|a_2| \approx 0.06$ when using the short-distance QCD corrected coefficients. An analogous effect is also known in nonleptonic D decays [10], and several authors advocated a modified procedure to evaluate the factorized amplitudes [10, 20]: There, only terms which are dominant in the $1/N$ expansion are taken into account. Recently there has been much discussion in the literature concerning these issues. Some authors have argued that QCD sum rules validate this procedure [21]. As our model for evaluating the matrix elements of the weak Hamiltonian we also

choose this leading $1/N$ approximation and use the QCD corrected coefficient functions $\bar{c}_i$ given above. We note that the terms proportional to $1/N$ in eq. (10) must then be dropped as well.

The strong phase shifts are generated in our model only by the absorptive parts (hard final state interactions) of the quark-level matrix elements of the effective Hamiltonian. Of course, when factorizing the hadronic matrix elements, all information on the crucial value of the momentum transfer k^2 of the gluon in the penguin diagram (fig. 1b) is lost. While it has been attempted [12] to model a more realistic momentum distribution by taking into account the exchange of a hard gluon, we will use here for simplicity only a fixed value of k^2. From simple two body kinematics [24] or from the investigations in ref. [12] one expects k^2 to be typically in the range

$$\frac{m_b^2}{4} \lesssim k^2 \lesssim \frac{m_b^2}{2} \ . \tag{16}$$

3 CP-violating observables

In general, CP-violating observables require the interference of two amplitudes with different weak phases, ϕ, from the CKM factors. To investigate the necessity of strong phases for the observables in $B \to VV$ decays (see section 1) and their interplay with the weak phase factors, we decompose the amplitudes into contributions proportional to v_u and v_c

$$A = v_u \cdot A^{(u)} + v_c \cdot A^{(c)} \ , \tag{17}$$

where A stands for a generic decay or helicity amplitude. The amplitudes for the CP conjugate process, A^{CP}, are then obtained from eq. (17) by replacing v_q by v_q^*.

Observables which involve only the real parts of the interfering amplitudes, like the decay rate or the parameters $\alpha_{1,2}$, can signal CP violation only when one compares them with the corresponding quantities of the charge conjugate decay channel, and when both, non-vanishing weak phase differences *and* strong phase shifts, $\delta_u - \delta_c$, are present. For instance, (defining $v_q \equiv |v_q| e^{i\phi_q}$)

$$\Gamma - \bar{\Gamma} \sim Im[v_u v_c^*] \cdot Im[A_u A_c^*] \sim \ \sin(\phi_u - \phi_c) \cdot \ \sin(\delta_u - \delta_c) \ . \tag{18}$$

On the other hand, the decay parameters β_i ($i = 1, 2$) can have non-zero values in the presence of either weak *or* strong phases alone. Then, by comparison with the parameters $\bar{\beta}_i$ (β_i^{CP}) of the C (CP) conjugate decay, one can, in principle, establish a weak phase difference even for vanishing strong phases

$$\beta_i + \bar{\beta}_i \ = \ \beta_i - \beta_i^{CP} \ \sim \ Im[v_u v_c^*] \cdot Re[A_u A_c^*] \ \sim \ \sin(\phi_u - \phi_c) \cdot \ \cos(\delta_u - \delta_c) \ . \tag{19}$$

or measure the strong phase shifts even for negligible weak phases

$$\beta_i - \bar{\beta}_i \;=\; \beta_i + \beta_i^{CP} \;\sim\; Re[v_j v_k^*] \cdot Im[A_j A_k^*] \;\sim\; \cos(\phi_j - \phi_k) \cdot \; \sin(\delta_j - \delta_k) \;, \tag{20}$$

where v_j and v_k are not necessarily different. In eqs. (19) and (20) we have dropped terms proportional to $Im[v_u v_c^*] \cdot Im[A_u A_c^*]$ which arise from the different denominators, Γ and $\bar{\Gamma}$, in β_i and $\bar{\beta}_i$. Note also the relative sign between $\bar{\beta}_i$ and $\beta_i^{\rm CP}$ due to the parity reflection.

If there are no CP-violating weak phases then $\beta_i = -\bar{\beta}_i$, $\alpha_i = \bar{\alpha}_i$, and $\Gamma = \bar{\Gamma}$, while the absence of strong phases implies $\beta_i = \bar{\beta}_i$ (and, of course, $\alpha_i = \bar{\alpha}_i$ and $\Gamma = \bar{\Gamma}$). Interesting CP differences in the case $B \to VV$, which do not require strong phases [see eq. (19)] and which are proportional to weak phase differences, are [using the phase definitions of eq. (14)]

$$Im(H_{+1}H_{-1}^* + \bar{H}_{+1}\bar{H}_{-1}^*) = -4\sqrt{x^2-1}\sum_{i,j} sin(\phi_{ai} - \phi_{cj})\; cos(\delta_{ai} - \delta_{cj})|a_i c_j| \tag{21}$$

and

$$\begin{aligned} Im(H_{+1}H_0^* - H_{-1}H_0^* + \bar{H}_{+1}\bar{H}_0^* - \bar{H}_{-1}\bar{H}_0^*) = & \\ -4(x^2-1)^{\frac{3}{2}}\sum_{i,j} sin(\phi_{ci} - \phi_{bj})\; cos(\delta_{ci} - \delta_{bj})|c_i b_j| & \\ -4x\sqrt{x^2-1}\sum_{i,j} sin(\phi_{ci} - \phi_{aj})\; cos(\delta_{ci} - \delta_{aj})|c_i a_j| & \quad . \end{aligned} \tag{22}$$

Here, $\bar{H}_\lambda$ are the amplitudes of the charge conjugated process and the subscripts of the weak and strong phases refer to different weak (or isospin, etc.) contributions.

The presence of strong phases is unambiguously demonstrated by a partial rate asymmetry as well as angular correlations of the following kind:

$$\frac{2\pi}{\Gamma}\frac{d\Gamma}{d\phi} - \frac{2\pi}{\bar{\Gamma}}\frac{d\bar{\Gamma}}{d\phi} = -(\alpha_2 - \bar{\alpha}_2)cos2\phi - (\beta_2 - \bar{\beta}_2)sin2\phi \tag{23}$$

Other terms can be isolated by examining the ϕ dependence of the differential rate difference between same hemisphere (SH) events (e.g. $0 < \theta_1, \theta_2 < \frac{\pi}{2}$) and opposite hemisphere (OH) events (e.g. $0 < \theta_1 < \frac{\pi}{2}$, $\frac{\pi}{2} < \theta_2 < \pi$):

$$\frac{2\pi}{\Gamma}(\frac{d\Gamma^{OH}}{d\phi} - \frac{d\Gamma^{SH}}{d\phi}) - \frac{2\pi}{\bar{\Gamma}}(\frac{d\bar{\Gamma}^{OH}}{d\phi} - \frac{d\bar{\Gamma}^{SH}}{d\phi}) = -\frac{1}{2}\{(\alpha_1 - \bar{\alpha}_1)cos\phi - (\beta_1 - \bar{\beta}_1)sin\phi\} \tag{24}$$

In general the dominant terms in the angular correlations are Γ_T/Γ, Γ_L/Γ, α_1 and α_2. The terms β_1 and β_2 are small and they are nonvanishing only if the helicity amplitudes H_{+1}, H_{-1} and H_0 or the invariant amplitudes a, b and c, respectively, have different phases. In all those channels where factorization is possible only in one way, this is not

the case because all matrix elements become simply proportional to each other; this is, of course, due to the simplicity of our model. For the same reason one has $\alpha_i = \bar{\alpha}_i$ (and $\Gamma_T/\Gamma = \bar{\Gamma}_T/\bar{\Gamma}$) in all the channels with vanishing β_i: The overall (weak and/or strong) phase factor cancels in the ratios that enter in the definition of the α_i [see (3)].

While the next-to-leading logarithmic precision of the effective Hamiltonian allows one to consistently calculate all amplitudes at order α_s and to include all one-loop matrix elements, some care is necessary when evaluating CP-violating asymmetries of the decay rates or of the observables of the angular distribution. In particular, one should make sure that the rate asymmetries for sufficiently inclusive channels remain consistent with CPT constraints in certain mass limits [6].

In order to specify a procedure which meets this requirement, we recall that absorptive parts, which arise from intermediate states having the *same* quark-gluon content as the final state, play a crucial role for CPT consistency of rate asymmetries in *inclusive* decays: It has been shown (see e.g. [23, 22]) that the inclusive rate difference due to absorptive parts from these "flavour-diagonal" interactions must, and in fact does, cancel if all interferences are taken into account which contribute at a given order in α_s (see appendix B for an example of such a "CPT cancellation"). Of course, this cancellation need not be generally true when the phase space of the final state is restricted or when exclusive decays are considered. Nevertheless, in our model for the *exclusive* amplitudes these CPT cancellations hold for many of the diagrams due to the factorization of the hadronic matrix elements. We shall assume that analogous CPT cancellations are — at least approximately — valid for all interferences of exclusive amplitudes to be evaluated here, and we therefore neglect absorptive parts from flavor diagonal rescattering throughout.

In this approximation all imaginary parts of the terms $\Delta F_1(k^2/m_q^2)$ are to be dropped in (11) and (12) when q refers to the flavor of a $q\bar{q}$-pair which is present in the final state. Moreover, no absorptive parts of vertex correction diagrams have to be evaluated, because they are always flavour diagonal. For consistency, the same procedure as for the rates should, of course, be systematically applied when evaluating the decay parameters (3) and their CP-differences. To get an idea of the quality of our approximation, we calculated the flavour-diagonal absorptive parts for the case of all penguin-like matrix elements, (11) and (12), and we found that their effect is indeed small.

In our calculation we do not explicitly drop higher order terms which arise, for instance, through interferences among (real and imaginary parts of) the order α_s matrix elements. However, such terms can not introduce the above mentioned inconsistencies with CPT because the flavour-diagonal absorptive parts are discarded. A completely systematic treatment of the higher order terms, some details of which we describe in

appendix C, would require to expand all *products* of interfering amplitudes — and not the amplitudes themselves — in term of α_s and the other couplings of the (effective) theory.

4 Results and Discussion

For a numerical analysis of the decay parameters and their CP-violating effects within our model, we need to specify the CKM matrix elements and the current form factors. It is well known [25] that fits for the parameters [4]

$$\begin{aligned} \rho &= cos\delta_{13}\ s_{13}/(s_{12}s_{23}) \\ \eta &= sin\delta_{13}\ s_{13}/(s_{12}s_{23}) \end{aligned} \tag{25}$$

of the CKM matrix depend critically on the value of the B-meson decay constant f_B. The solution for lower f_B values leads to a negative ρ while higher f_B values render ρ positive.

We have calculated our results for the positive ρ solution, with the values

$$\rho = 0.32\ , \quad \eta = 0.31$$

(i.e. $s_{13} = 0.0045$, $\delta_{13} = 44\,\mathrm{deg}$) from the analysis by Schmidtler and Schubert[25] for $f_B = 250$ MeV (giving $m_t = 135 \pm 27$ GeV). A more recent analysis by Ali and London [27] based on the latest information on V_{ub} yields similar results. For comparison, we will also show the asymmetries calculated with the negative ρ solution: $\rho = -0.41$ and $\eta = 0.18$ [25] corresponding to $f_B = 125$ MeV (giving $m_t = 172 \pm 15$ GeV).

The main purpose of this work is to calculate the effect of (perturbative) strong phases on the angular correlation coefficients obtained earlier with the strong phase put to zero [8]. Moreover, we have included pure penguin channels and calculated partial rate asymmetries. (The pure penguin modes are $B^- \to \rho^- K^{*0}$, $K^{*-}\Phi$, $\rho^-\Phi$ and $B^- \to K^{*-}K^{*0}$.) Of course, the estimates of the CP-violating observables given here may suffer from large uncertainties due to strong phases from soft final state interactions and, therefore, can at most be indicative of the expected orders of magnitude.

First we consider the results for the ρ positive case (tables 1 and 2). To see the effect of strong phases we also show in the following tables the results without strong phases from imaginary parts of the one-loop matrix elements (the values are given in parentheses and only where different from those with strong phases included). These numbers may differ from the results in [7, 8] since we include here throughout the order

[4] These coincide with the parameters ρ and η of the Wolfenstein representation for small angles [26].

α_s real parts of the penguin-like one-loop matrix elements. The branching ratios are calculated using $\tau_B = 1.49\,\mathrm{p\,sec}$, and a dash (—) in the tables indicates values which must be exactly zero in our model.

The parameters β_1 and β_2 of the azimuthal decay distribution are only non-zero for the decays $B^- \to K^{*-}\omega$, $K^{*-}\rho^0$ and $\rho^-\omega$. The other decays have $\beta_i = 0$ in our model because all three helicity amplitudes have the same overall (combined weak and strong) phase. This is always the case when there is only one way to factorize the hadronic matrix element, e.g. in $B^- \to K^{*-}J/\Psi$, $D_s^{*-}D^{*0}$, etc., and in all pure penguin modes. A special case is $B^- \to \rho^0\rho^-$: if isospin breaking (due to the mass difference between ρ^- and ρ^0) is neglected, no penguin contributions are present[5] and, hence, no weak phase differences can occur. Concerning the CP-violating effects in the angular distributions the most promising decays in the case of vanishing strong phases (values in parentheses) are $B^- \to K^{*-}\omega$ and $B^- \to K^{*-}\rho^0$: They have branching ratios of the order $(1\text{–}3)\times 10^{-6}$ and $|\beta_1| \approx (1\text{–}4)\times 10^{-2}$ whereas the β_2 are a factor of ten smaller. (See table 1.)

Table 1 also illustrates the influence of the strong phases generated by the absorptive parts of the matrix elements. They have two effects: First, they produce the rate asymmetry $a_{\mathrm{CP}} \neq 0$, which is given in the third column, and second, they generate different angular distributions for certain charge-conjugate decays, i.e. $\alpha_i \neq \bar{\alpha}_i$ and $\beta_i \neq \bar{\beta}_i$.

The rate asymmetry a_{CP} is appreciable in some of the cases, e.g. for $B^- \to K^{*-}\omega$ ($a_{\mathrm{CP}} \approx 28\%$) and $B^- \to K^{*-}\rho^0$ ($a_{\mathrm{CP}} \approx 15\%$). Both decays have approximately the same branching ratio of the order $(2\text{–}4)\times 10^{-6}$. For the pure penguin modes these asymmetries are either smaller ($\approx 1\%$) or, as in the case of $B^- \to K^{*-}K^{*0}$, the branching ratios are tiny ($\sim 10^{-7}$). Interesting is the decay $B^- \to D^{*-}D^{*0}$ with a branching ratio of 0.1% and a rate asymmetry[6] of about 1%. For $B^- \to \rho^-K^{*0}$ the branching ratio and a_{CP} are similar to the results for π^-K^{*0} obtained recently by Fleischer [28]

For β_1 and β_2 we observe a significant effect from the strong phase shifts for the most interesting case $B^- \to K^{*-}\omega$. From our results for $\bar{\beta}_1$ and $\bar{\beta}_2$ for the charge conjugate decay $B^+ \to K^{*+}\omega$ (see the corresponding line in table 1) we find that $\beta_i + \bar{\beta}_i$ is not drastically changed as compared to the case with no strong phases. We obtained from table 1 $(\beta_1 + \bar{\beta}_1)/2 = -29 \times 10^{-3}$ and $(\beta_2 + \bar{\beta}_2)/2 = 2.8 \times 10^{-3}$ which is to be compared to $\beta_1 = \bar{\beta}_1 = -37 \times 10^{-3}$ and $\beta_2 = \bar{\beta}_2 = 3.6 \times 10^{-3}$. On the other hand, we find $(\beta_1 - \bar{\beta}_1)/2 = -14 \times 10^{-3}$ and $(\beta_2 - \bar{\beta}_2)/2 = 1.4 \times 10^{-3}$ which shows the effect of the strong phases [see eq. (16) and eq.(17)]. In the case of $K^{*-}\rho^0$ the behavior is less significant, e.g. $(\beta_1 + \bar{\beta}_1)/2 = 6.7 \times 10^{-3}$ and $(\beta_1 - \bar{\beta}_1)/2 = -0.4 \times 10^{-3}$ (compared to

[5] This has been missed in [8].

[6] This channel is rather insensitive to the particular choice of k^2 because the leading absorptive part comes from a $u\bar{u}$-cut with a low threshold.

$\beta_1 = \bar{\beta}_1 = 7.2 \times 10^{-3}$ without strong phases). In the $\rho\omega$ channel the pattern is even more pronounced: while the $\beta_i + \bar{\beta}_i$ are again not changed substantially by the strong phases, the differences, $(\beta_1 - \bar{\beta}_1)/2 = 0.16 \times 10^{-3}$ and $(\beta_2 - \bar{\beta}_2)/2 = -0.002 \times 10^{-3}$, are now larger than the sums.

In the analogous calculations for $\alpha_i - \bar{\alpha}_i$ (and $\Gamma_T/\Gamma - \bar{\Gamma}_T/\bar{\Gamma}$) we find significant C-differences only in the case of $K^{*-}\omega$, where the relative differences of Γ_T/Γ, α_1 and α_2 are of the order of several percent (detailed numbers can be extracted from table 1). In the two other cases, $K^{*-}\rho^0$ and $\rho^-\omega$, these differences are smaller.

We have also performed the corresponding calculation with the $1/N$ terms included throughout (see table 2). As a result, some of the predicted branching ratios (BR) change drastically; for instance, BR($B^- \to K^{*-} J/\Psi$) is decreased from 3.8×10^{-3} to 1.6×10^{-4}. Since the average experimental value of this branching ratio is $(0.17 \pm 0.05)\%$ [19] the version without $1/N$ terms is to be preferred in this decay. The pure penguin modes of the $b \to s$ transitions are rather insensitive to the inclusion of the $1/N$ terms with the exception of $K^*\Phi$ whose rate is increased by a factor of three. The branching ratios for all other $b \to s$ transitions in the upper part of the tables remain more or less unchanged, while β_1 for $B^- \to K^{*-}\omega$ and $B^- \to K^{*-}\rho^0$ become smaller and look less interesting. The pattern for $b \to d$ transitions (see the lower part of table 2) is different when compared with table 1: the branching ratio for the pure penguin mode $B^- \to \rho^-\Phi$ is drastically reduced and also the rate for $B^- \to \rho^- J/\Psi$ becomes more than an order of magnitude smaller.

It is obvious that the color unsuppressed decays $B^- \to D_s^{*-} D^{*0}$, $D^{*0} D^{*-}$ (with amplitudes proportional to $\bar{c}_2 + \bar{c}_1/N$) are not significantly influenced by the treatment of the $1/N$ terms. We should also note that the coefficients Γ_T/Γ, Γ_L/Γ, α_1 and α_2, in the angular distribution are not very sensitive to the treatment of the $1/N$ terms and to assumptions about the strong and weak phases. They depend mainly on the helicity structure of the matrix elements and they are therefore more important for testing the underlying model assumptions, in particular, the current matrix elements in the BSW model in conjunction with the factorization hypothesis.

To give an impression how sensitive our results are with respect to the solution ambiguity in the CKM parameter determination we have repeated the calculations with ρ negative. The results (without 1/N terms) are shown in table 3 and should be compared with the numbers in table 1 (with strong phases). Generally the branching ratios and asymmetries are similar in magnitude. In the interesting $K^{*-}\omega$ and $K^{*-}\rho^0$ final states the branching ratios decrease or increase, respectively, while the rate asymmetries and azimuthal asymmetries vary in the opposite direction. The pure penguin modes change less but in a similar way. Since the charge asymmetries of the various observables can

be more pronounced for a ρ negative CKM matrix, the interesting charge conjugate channels are also included in table 3.

We mention that some of the pure penguin modes have been calculated by other authors. Davies et al. obtained a comparable result for the partial rate of $B \to K^*\Phi$ [29] using only leading logarithmic order QCD coefficients. Dong-sheng Du et al. [30] calculated the $\rho\Phi$ rate using also coefficients of Buras et al.. They obtained similar results, in particular also the strong suppression of this rate when 1/N terms are included.

In this work we have confined our attention to direct CP violation in decays of charged B mesons. The strong phases also contribute to some *neutral* B decays, where their effect on the CP-violating time dependence is a further complication in the analysis of CP violation arising from the interference of mixing and decay amplitudes. This subject is currently under investigation.

Finally, we would like to note that the partial rate asymmetries we have found are larger than those reported in B decays to two pseudoscalar mesons, making $B \to VV$ an attractive channel for probes of direct CP violation.

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Appendix A: Matrix elements

The momentum dependence of the penguin-like matrix elements entering in (11) and (12) is given by

$$\begin{aligned} \Delta F_1(z) &= -4\int_0^1 u(1-u)\ell n\left[1-zu(1-u)\right]du \\ &= \frac{2}{3}\left\{\frac{5}{3}+\frac{4}{z}+(1+\frac{2}{z})R(z)\right\}\ , \end{aligned} \tag{26}$$

where, setting $r \equiv \sqrt{|1-4/z|}$,

$$R(z) = \begin{cases} r\cdot \ell n\frac{r-1}{r+1} & (z<0) \\ -2+\frac{z}{6}+\frac{z^2}{60}+\frac{z^3}{420}+\cdots & (z\to 0) \\ -r\pi + 2r\arctan r & (0<z<4) \\ +ir\,\pi + r\ell n\frac{1-r}{1+r} & (z>4) \end{cases}$$

For completness, we list here also the factorized matrix elements of $\mathcal{H}_{\rm eff}$ for the pure penguin modes not yet presented in former publications [8]:

$$\langle \rho^- K^{*0}|\mathcal{H}^{(q)}_{\rm eff}|B^-\rangle = v_q\sqrt{2}G_F\Big(\frac{1}{N}c_3^{\rm eff}+c_4^{\rm eff}\Big)\langle\rho^-|\bar d\gamma_\mu b_L|B^-\rangle\langle K^{*0}|\bar s\gamma^\mu d|0\rangle\ , \tag{27}$$

$$\begin{aligned}\langle K^{*-}\Phi|\mathcal{H}^{(q)}_{\rm eff}|B^-\rangle &= v_q\sqrt{2}G_F\Big((1+\frac{1}{N})c_3^{\rm eff}+(1+\frac{1}{N})c_4^{\rm eff}+c_5^{\rm eff}+\frac{1}{N}c_6^{\rm eff}\Big) \\ &\quad \cdot\langle K^{*-}|\bar s\gamma_\mu b_L|B^-\rangle\langle\Phi|\bar s\gamma^\mu s|0\rangle\ ,\end{aligned} \tag{28}$$

$$\begin{aligned}\langle \rho^-\Phi|\mathcal{H}^{(q)}_{\rm eff}|B^-\rangle &= v_q\sqrt{2}G_F\Big(c_3^{\rm eff}+\frac{1}{N}c_4^{\rm eff}+c_5^{\rm eff}+\frac{1}{N}c_6^{\rm eff}\Big) \\ &\quad \cdot\langle \rho^-|\bar d\gamma_\mu b_L|B^-\rangle\langle\Phi|\bar s\gamma^\mu s|0\rangle\ ,\end{aligned} \tag{29}$$

$$\langle K^{*-}K^{*0}|\mathcal{H}^{(q)}_{\rm eff}|B^-\rangle = v_q\sqrt{2}G_F\Big(\frac{1}{N}c_3^{\rm eff}+c_4^{\rm eff}\Big)\langle K^{*-}|\bar s\gamma_\mu b_L|B^-\rangle\langle K^{*0}|\bar d\gamma^\mu s|0\rangle\ . \tag{30}$$

Appendix B: CPT cancellations in inclusive rate asymmetries

In this appendix we shall illustrate a typical example of a "CPT cancellation" for the CP-violating rate difference in inclusive decays. In contrast to the discussion within the full theory of refs. [6, 22] we adopt here the framework of the effective Hamiltonian where one can perform a completely analogous analysis. To render the correspondence to diagrams in the full theory more obvious, we assume the simplified situation where all $\bar c_i$ are zero, except $\bar c_2$.

An illustrating channel is the charmless $\Delta b = \Delta s = 1$ transition $b \to su\bar u$: At order α_s the rate difference $a_{CP} \sim \Gamma - \bar\Gamma$ can arise only from the interference between

the tree-level matrix element of $O_2^{(u)}$ and the penguin diagram of $O_2^{(c)}$ (see fig. 2). The resulting asymmetry is small because the absorptive part of the penguin is kinematically suppressed by the $c\bar{c}$-threshold. At order α_s^2 one finds two types of contributions: First, the interference between penguin-like matrix elements of $O_2^{(u)}$ and $O_2^{(c)}$ (see fig. 3a), and second, interferences between the tree level matrix element of $O_2^{(u)}$ and order g_s^4 diagrams with an insertion of $O_2^{(c)}$. In particular, there is the interference with a penguin-like diagram of $O_2^{(c)}$ having an additional vacuum polarization on the gluon line (fig. 3b). An absorptive part of this diagram can be generated through a $u\bar{u}$-pair inside the loop. In this case, the two order α_s^2 interferences depicted in fig. 3 differ only by interchanging the role of the final and the "cut" state, which generates the absorptive part. Applying the Cutkosky rules one readily finds (see also [23, 22]) that both interferences have the same size, and their combined effect cancels due to the relative minus sign in

$$\Gamma - \bar{\Gamma} \sim Im[A_u A_c^*] = Im[A_u] \cdot Re[A_c] - Re[A_u] \cdot Im[A_c] \; .$$

Note that the cancellation between the diagrams of, for instance, fig. 3 works also for the (factorized!) *exclusive* amplitudes because the quark loop contributes in both cases the same multiplicative factor.

Appendix C: Systematic expansion of the observables

In the framework of the effective Hamiltonian described in section 2, the natural choice for the couplings, in terms of which the observables are to be expanded, is α_s at $\mu = m_b$ together with the manifestly renormalization scheme independent coefficients $\bar{c}_i$. Of course, CPT cancellations analogous to the ones discussed above are present for each interference proportional to $\bar{c}_i\bar{c}_j\alpha_s^n$ with $n \geq 2$. In table 4 we have specified the order up to which our calculation of the various observables is complete. Here, we use c_t and c_p as generic notations for any of the coefficients $\{\bar{c}_1, \bar{c}_2\}$ of $O_{1,2}$ or of the coefficients $\{\bar{c}_3, \ldots, \bar{c}_6\}$ of the penguin operators, respectively [not necessarily in the combinations defined in (11) and (12)].

For decay modes with tree and penguin contributions we do not cover any terms of order α_s^2 in a complete way. However, since we are counting the powers of $\alpha_s(m_b)$ within the *effective* theory, the interference between local penguin contributions ($\sim c_p$) and absorptive penguin-like matrix elements ($\sim \alpha_s c_t$) is included here already at order α_s (while it is order α_s^2 in the full theory). For instance, the local parts of the subdiagrams in fig. 3 (viewed as diagrams in the full theory), which are enclosed by a box, are taken into account in the effective theory through the penguin operators $O_3, \ldots, O_6$. Although the coefficients c_p, being implicitly of order $\alpha_s(M_W)$, are small, the order $\alpha_s c_t c_p$ interferences can be numerically important for the rate asymmetries in some charmless decays where

the $\alpha_s c_t^2$ contributions (see e.g. Fig. 2) are kinematically suppressed due to the $c\bar{c}$-threshold [6].

In the case of pure penguin modes, i.e. final states which have at tree level only contributions from the penguin operators $O_3, \ldots, O_6$, interferences of order $\alpha_s^2 c_t^2$ can arise only as a product of two order g_s^2 amplitudes. Therefore, the penguin-like one-loop matrix elements are sufficient for the complete treatment of these interferences. On the other hand, we may neglect systematically all terms of order $\alpha_s c_p^2$ and $\alpha_s^2 c_t c_p$. Thus, we retain effectively the terms which originate from order α_s^2 contributions in the full theory (and before the renormalization group evolution which sums up powers of $\alpha_s \times \ell n \mu^2/M_W^2$). Of course, this procedure is also numerically sensible, because $c_t \gg c_p$ by at least a factor of five ($\approx 1/\alpha_s$) due to their origin from penguin diagrams.

At any order higher than the ones listed in table 4, real parts of vertex-correction diagrams are necessary for a complete treatment. Their imaginary parts, which enter already in the terms enclosed by parentheses in the table, correspond to flavour-diagonal rescattering and are neglected throughout our calculations. Of course, a complete treatment of all order α_s^2 interferences also requires one to calculate a large number of two-loop diagrams [22] for the matrix elements of $\mathcal{H}_{\rm eff}$. In fact, the next-to-leading logarithmic precision for the coefficient functions, just allows one to evaluate their absorptive parts (but not their real parts) without encountering new renormalization scheme dependencies.

References

[1] *CP Violation*, Editor C. Jarlskog, Advanced Series on Directions in High Energy Physics, Vol. 3, World Scientific, Singapore, New Jersey, London, Hong Kong, 1989.

[2] J. H. Christenson, J. W. Cronin, V. L. Fitch, R. Turlay, Phys. Rev. Lett. 13 (1964) 138.

[3] I.I. Bigi et al. in CP Violation, Editor C. Jarlskog, Advanced Series on Directions in High Energy Physics, Vol. 3, World Scientific, Singapore, New Jersey, London, Hong Kong, 1989 and the earlier literature quoted there.

[4] H. Burkhardt et al., Phys. Lett. B206 (1985) 169
J. R. Patterson et al., Phys. Rev. Lett. 64 (1990) 1491.

[5] M. Bander, D. Silverman, A. Soni, Phys. Rev. Lett. 43 (1979) 242.

[6] J. M. Gerard, W. S. Hou, Phys. Rev. Lett 62 (1989) 855, Phys. Rev. D43 (1991) 2909.

[7] G. Valencia, Phys. Rev. D39 (1989) 3339.

[8] G. Kramer and W. Palmer, Phys. Rev D 45 (1992) 132, Phys. Lett. B279 (1992) 181, Phys. Rev. D46 (1992) 2969.

[9] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.

[10] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C34 (1987) 103, *ibid* C29 (1985) 637.

[11] S.J. Brodsky and P. Lepage. Phys. Lett. B87 (1979) 359 and Phys. Rev. D22 (1980) 2157; A. Szczepaniak, E.M. Henley and S.J. Brodsky, Phys. Lett. B243 (1990) 287.

[12] H. Simma and D. Wyler, Phys. Lett. B272 (1991) 395.

[13] R. Fleischer, Z. Phys. C58 (1993) 483.

[14] For complete joint angular distributions including bottom baryon decays see P. Bialas et. al., Z. Phys. C57 (1993) 115.

[15] A. Buras et al., Nucl. Phys. B370 (1992) 69.

[16] F. Gilman and M. Wise, Phys. Rev. D20 (1979) 2392,
W. Ponce, Phys. Rev. D23 (1981) 1134.

[17] G. Altarelli in Proceedings of *QCD 20 Years Later*, edited by P.M. Zerwas and H. A. Kastrup, Aarchen, June 9–13, 1992 World Scientific, Singapore, New Jersey, London, Hong Kong, 1993.

[18] D. Fakirov and B. Stech, Nucl. Phys. B133 (1978) 315, N. Cabibbo and L. Maiani, Phys. Lett. 73B (1978) 418 and earlier references given there.

[19] H. Albrecht (ARGUS), et al., Z. Phys. C48 (1990) 543, D. Bortoletto et. al. (CLEO), Phys. Rev. D45 (1992) 21.

[20] A. J. Buras, J.-M. Gerard and R. Rückl, Nucl. Phys. B268 (1986) 16; W. A. Bardeen, A. J. Buras and J.-M. Gerard, Phys. Lett. B180 (1980) 133.

[21] B. Blok and M. Shifman, Sov. Journ. Nucl. Phys. 45 (1987) 135,301,522.

[22] J. M. Gerard and W.-S. Hou, Phys. Rev. D43 (1991) 2909; H. Simma, G. Eilam and D. Wyler, Nucl. Phys. B352 (1991) 367

[23] L. Wolfenstein, Santa Barbara preprint NSF-ITP-90-29, and Phys. Rev. D43 (1991) 151.

[24] N.G. Deshpande and J. Trampetic, Phys. Rev. D41 (1990) 2926.

[25] M. Schmidtler and K. Schubert, Z. Physik C 53 (1992) 347.

[26] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.

[27] A. Ali, D. London in ECFA Workshop on a European *B*-Meson Factory, *B* Physics Working Group Report. Editors R. Aleksan (Saclay) and A. Ali (DESY), ECFA 93/151, DESY 93-053, p. 42-68.

[28] R. Fleischer, preprint, TU München, TUM-T31-40/93.

[29] A. J. Davies et al., preprint AEU-02-93, EHU-03-93, KGU-02-93.

[30] Dong-sheng Du, Zhi-zhong Xing, preprint, Institute of High Energy Physics, Beijing.

Figure captions

Fig. 1: Two types of one-loop matrix elements: (a) Vertex corrections, and (b) penguin diagrams. The square box denotes an insertion of one of the four-quark operators O_i of eq. (5).

Fig. 2: Two interfering diagrams generating a rate asymmetry at order α_s. The absorptive phase arises from the cut state indicated by the dashed line.

Fig. 3: Examples of order α_s^2 interferences: (a) Two $O(g_s^2)$ penguin diagrams, and (b) a tree and a $O(g_s^4)$ diagram. The rate differences due to the absorptive parts from the $\bar{u}u$ cut (dashed line) cancel. The local part of the subdiagrams enclosed by the dotted box is taken into account by penguin operators in the effective theory.

Table captions

Tab. 1: Branching ratios, rate asymmetries and angular correlation coefficients, using matrix elements *without* $1/N$ Terms for a ρ *positive* CKM Matrix ($\rho = 0.32, \eta = 0.31$). Values in parentheses correspond to the case without strong phases.

Tab. 2: Branching ratios, rate asymmetries and angular correlation coefficients, using matrix elements *with* $1/N$ Terms for a ρ *positive* CKM Matrix ($\rho = 0.32, \eta = 0.31$). Values in parentheses correspond to the case without strong phases.

Tab. 3: Branching ratios, rate asymmetries and angular correlation coefficients, using matrix elements *without* $1/N$ Terms for a ρ *negative* CKM Matrix ($\rho = -0.41, \eta = 0.18$).

Tab. 4: Orders of α_s, c_t and c_p at which the treatment of the various observables is complete (neglecting absorptive parts from flavour-diagonal rescattering in vertex-correction diagrams).

Table 1

Matrix Elements *without* $1/N$ Terms and *with (without)* Strong Phases ρ *positive* CKM Matrix: $\rho = 0.32, \eta = 0.31$							
Channel	BR	a_{CP} [%]	$\frac{\Gamma_T}{\Gamma}$	α_1	α_2	β_1 $[10^{-3}]$	β_2 $[10^{-3}]$
$b \to s$ transitions: $\Delta c = 0,\ \Delta b = \Delta s = 1$							
$B^- \to \rho^- + K^{*0}$	$1.3{\times}10^{-5}$	0.54	0.107	-0.334	0.009	—	—
	$(1.2{\times}10^{-5})$	(—)					
$B^- \to K^{*-} + \Phi$	$5.5{\times}10^{-6}$	1.22	0.137	-0.385	0.017	—	—
	$(4.3{\times}10^{-6})$	(—)					
$B^- \to K^{*-} + \omega$	$2.4{\times}10^{-6}$	28	0.089	-0.316	0.011	-44	4.2
$B^+ \to K^{*+} + \omega$	$1.3{\times}10^{-6}$		0.073	-0.300	0.012	-15	1.5
	$(1.7{\times}10^{-6})$	(—)	(0.080)	(-0.307)	(0.012)	(-37)	(3.6)
$B^- \to K^{*-} + \rho^0$	$3.7{\times}10^{-6}$	15	0.105	-0.332	0.009	7.1	-0.63
$B^+ \to K^{*+} + \rho^0$	$2.7{\times}10^{-6}$		0.103	-0.330	0.009	6.3	-0.55
	$(3.0{\times}10^{-6})$	(—)	(0.104)	(-0.331)	(0.009)	(7.2)	(-0.64)
$B^- \to K^{*-} + J/\Psi$	$3.8{\times}10^{-3}$	—	0.427	-0.621	0.123	—	—
	$(3.8{\times}10^{-3})$	(—)					
$B^- \to D_s^{*-} + D^{*0}$	$2.4{\times}10^{-2}$	-0.05	0.476	-0.664	0.183	—	—
	$(2.4{\times}10^{-2})$	(—)					
$b \to d$ transitions: $\Delta s = \Delta c = 0,\ \Delta b = 1$							
$B^- \to \rho^- + \Phi$	$1.1{\times}10^{-7}$	—	0.130	-0.364	0.011	—	—
	$(1.1{\times}10^{-7})$	(—)					
$B^- \to K^{*-} + K^{*0}$	$4.2{\times}10^{-7}$	-18	0.112	-0.352	0.014	—	—
	$(4.8{\times}10^{-7})$	(—)					
$B^- \to \rho^- + \omega$	$1.3{\times}10^{-5}$	-6.6	0.083	-0.298	0.007	$+0.22$	-0.003
$B^+ \to \rho^+ + \omega$	$1.5{\times}10^{-5}$		0.083	-0.298	0.007	-0.10	$+0.001$
	$(1.4{\times}10^{-5})$	(—)				(0.05)	(-0.001)
$B^- \to \rho^- + \rho^0$	$1.3{\times}10^{-5}$	—	0.083	-0.298	0.007	—	—
	$(1.3{\times}10^{-5})$	(—)					
$B^- \to \rho^- + J/\Psi$	$1.8{\times}10^{-4}$	—	0.388	-0.597	0.097	—	—
	$(1.8{\times}10^{-4})$	(—)					
$B^- \to D^{*0} + D^{*-}$	$1.2{\times}10^{-3}$	0.89	0.456	-0.660	0.172	—	—
	$(1.2{\times}10^{-3})$	(—)					

Table 2

Matrix Elements *with* $1/N$ Terms and *with (without)* Strong Phases ρ *positive* CKM Matrix: $\rho = 0.32, \eta = 0.31$							
Channel	BR	a_{CP} [%]	$\frac{\Gamma_T}{\Gamma}$	α_1	α_2	β_1 $[10^{-3}]$	β_2 $[10^{-3}]$
$b \to s$ transitions: $\Delta c = 0,\ \Delta b = \Delta s = 1$							
$B^- \to \rho^- + K^{*0}$	1.0×10^{-5} (9.3×10^{-6})	0.56 (—)	0.107	-0.334	0.009	—	—
$B^- \to K^{*-} + \Phi$	1.5×10^{-5} (1.4×10^{-5})	0.56 (—)	0.137	-0.385	0.017	—	—
$B^- \to K^{*-} + \omega$	2.6×10^{-6} (1.8×10^{-6})	29 (—)	0.107 (0.107)	-0.334 (-0.333)	0.009 (0.009)	-0.95 (-1.2)	0.09 (0.12)
$B^- \to K^{*-} + \rho^0$	2.5×10^{-6} (1.8×10^{-6})	30 (—)	0.106 (0.106)	-0.333 (-0.333)	0.009 (0.009)	-1.6 (-1.9)	0.14 (0.17)
$B^- \to K^{*-} + J/\Psi$	1.6×10^{-4} (1.6×10^{-4})	— (—)	0.427	-0.621	0.123	—	—
$B^- \to D_s^{*-} + D^{*0}$	2.0×10^{-2} (2.0×10^{-2})	-0.05 (—)	0.476	-0.664	0.183	—	—
$b \to d$ transitions: $\Delta s = \Delta c = 0,\ \Delta b = 1$							
$B^- \to \rho^- + \Phi$	1.6×10^{-11} (1.6×10^{-11})	— (—)	0.130	-0.364	0.011	—	—
$B^- \to K^{*-} + K^{*0}$	3.1×10^{-7} (3.6×10^{-7})	-18 (—)	0.112	-0.352	0.014	—	—
$B^- \to \rho^- + \omega$	2.5×10^{-5} (2.6×10^{-5})	-4.1 (—)	0.084	-0.299	0.007	0.23 (0.17)	-0.003 (-0.002)
$B^- \to \rho^- + \rho^0$	2.3×10^{-5} (2.3×10^{-5})	— (—)	0.083	-0.298	0.007	—	—
$B^- \to \rho^- + J/\Psi$	7.0×10^{-6} (7.0×10^{-6})	— (—)	0.388	-0.597	0.097	—	—
$B^- \to D^{*0} + D^{*-}$	1.0×10^{-3} (1.0×10^{-3})	0.88 (—)	0.456	-0.660	0.172	—	—

Table 3

Matrix Elements *without* $1/N$ Terms and *with* Strong Phases ρ *negative* CKM Matrix: $\rho = -0.41, \eta = 0.18$							
Channel	BR	a_{CP} [%]	$\frac{\Gamma_T}{\Gamma}$	α_1	α_2	β_1 $[10^{-3}]$	β_2 $[10^{-3}]$
$b \to s$ transitions: $\Delta c = 0,\ \Delta b = \Delta s = 1$							
$B^- \to \rho^- + K^{*0}$	$1.3{\times}10^{-5}$	0.33	0.107	-0.334	0.009	—	—
$B^- \to K^{*-} + \Phi$	$5.2{\times}10^{-6}$	0.75	0.137	-0.385	0.017	—	—
$B^- \to K^{*-} + \omega$	$6.5{\times}10^{-7}$	83	0.135	-0.342	$+0.004$	-131	-13
$B^+ \to K^{*+} + \omega$	$6.0{\times}10^{-8}$		0.386	-0.402	-0.047	$+231$	-22
$B^- \to K^{*-} + \rho^0$	$9.0{\times}10^{-6}$	3.1	0.109	-0.335	0.009	$+0.74$	-0.06
$B^+ \to K^{*+} + \rho^0$	$8.5{\times}10^{-6}$		0.109	-0.335	0.009	$+2.18$	-0.19
$B^- \to K^{*-} + J/\Psi$	$3.9{\times}10^{-3}$	—	0.427	-0.621	0.123	—	—
$B^- \to D_s^{*-} + D^{*0}$	$2.4{\times}10^{-2}$	-0.03	0.476	-0.664	0.183	—	—
$b \to d$ transitions: $\Delta s = \Delta c = 0,\ \Delta b = 1$							
$B^- \to \rho^- + \Phi$	$3.9{\times}10^{-7}$	—	0.130	-0.364	0.011	—	—
$B^- \to K^{*-} + K^{*0}$	$1.4{\times}10^{-6}$	-3.5	0.112	-0.352	0.014	—	—
$B^- \to \rho^- + \omega$	$9.1{\times}10^{-6}$	-5.6	0.083	-0.298	0.007	-0.12	$+0.001$
$B^+ \to \rho^+ + \omega$	$10.{\times}10^{-6}$		0.083	-0.298	0.007	$+0.18$	-0.002
$B^- \to \rho^- + \rho^0$	$1.3{\times}10^{-5}$	—	0.083	-0.298	0.007	—	—
$B^- \to \rho^- + J/\Psi$	$1.6{\times}10^{-4}$	—	0.388	-0.597	0.097	—	—
$B^- \to D^{*0} + D^{*-}$	$1.2{\times}10^{-3}$	0.55	0.456	-0.660	0.172	—	—

Table 4

Observables	Decay mode	
	tree & penguin	pure penguin
$\Gamma,\ \alpha_i$	$c_t^2\,,\ c_t c_p\,,\ c_p^2$	$c_p^2\,,\ \alpha_s c_t c_p\,,\ \alpha_s^2 c_t^2$
$\Gamma - \bar{\Gamma}\,,\ \alpha_i - \bar{\alpha}_i$	$\alpha_s c_t^2\,,\ (\alpha_s c_t c_p\,,\ \alpha_s c_p^2)$	$\alpha_s c_t c_p\,,\ (\alpha_s c_p^2)\,,\ \alpha_s^2 c_t^2$
β_i	$c_t c_p\,,\ c_p^2\,,\ \alpha_s c_t^2$	$c_p^2\,,\ \alpha_s c_t c_p\,,\ \alpha_s^2 c_t^2$
$\beta_i - \bar{\beta}_i$	$(\alpha_s c_t^2\,,\ \alpha_s c_t c_p\,,\ \alpha_s c_p^2)$	$\alpha_s c_t c_p\,,\ (\alpha_s c_p^2)\,,\ \alpha_s^2 c_t^2$